Onset of Double Diffusive Convection in a Thermally Modulated Fluid-Saturated Porous Medium

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The onset of double diffusive convection in a sparsely packed porous medium was studied under modulated temperature at the boundaries, and a linear stability analysis has been made. The primary temperature field between the walls of the porous layer consisted of a steady part and a time-dependent periodic part and the Galerkin method and the Floquet were used. The critical Rayleigh number was found to be a function of frequency and amplitude of modulation, Prandtl number, porous parameter, diffusivity ratio and solute Rayleigh number.

Key words: Double Diffusive Convection; Thermal Modulation; Rayleigh Number; Porous Medium.

1. Introduction

Double diffusive convection in a porous medium [1] is of great importance as it finds its applications in many practical fields such as in petroleum industry [2], in extraction of geothermal energy [3 – 6], in engineering, e. g. alloy solidification process [7], and in geophysics [8]. The first study on the double diffusive generalization of the Horton-Rogers-Lapwood problem [9, 10] was given by Nield [11]. Thereafter literature has accumulated steadily, and there is a plethora of literature covering the situations of double diffusive convection in porous media. An excellent review of most of these studies has been given by Nield and Bejan [12]. Some other available studies using different physical models and boundary conditions are; Taunton and Lightfoot [13], Patil and Rudraiah [14], Griffith [15], Patil and Vaidyanathan [16], Poulikakos [17], Rudraiah and Malashetty [18], Murray and Chen [19], Patil et al. [20], Mohamad and Bennacer [21], Bennacer et al. [22], Oueslati et al. [23].

However most of these studies on double diffusive convection are mainly concerned with the convection in a horizontal porous layer heated and salted from below, i. e. the temperature gradient is a function only of the vertical coordinate z. This is not so common in real life problems, but one can find situations in which the temperature gradient is a function of both space and time. This temperature gradient (temperature modulation) can be determined by solving the energy equation with suitable thermal conditions, and can be used as a mechanism to control the convective flow. For example it can be used to control the quality and structure of the resulting solid by influencing the transport process during solidification of an alloy.

The first study on the effect of temperature modulation on the thermal instability in a horizontal fluid layer was carried out by Venezian [24]. Some other studies related to the modulation effect on the thermal instability in a fluid layer are also available. But studies on the convection in porous media with modulated temperature at the boundaries have received only limited attention. Some of the papers on this problem with a single component fluid are from Catlagirone [25], Chhuon and Catlagirone [26], Rudraiah and Malashetty [27, 28], Malashetty and Wadi [29], Malashetty and Basavaraja [30, 31] and Bhadauria [32]. However the literature on double diffusive convection in a porous medium with temperature modulation is scarce. Only recently Malashetty and Basavaraja [33] and Bhadauria [34] have investigated the problem of double diffusive convection in a porous layer using temperature modulation of free-free boundaries. Since free-free surfaces are less accessible to experiments, it is more appropriate to consider rigid-rigid boundaries. To the best of our knowledge no other literature is available on double diffusive convection in a porous layer in which temperature modulation of rigid boundaries has been considered.

Therefore in the present study the authors investigate the effect of temperature modulation on double diffusive convection in a horizontal porous layer for rigid-rigid boundaries. Brinkman's model has been considered to model the no-slip condition at these boundaries. The results reported in this study bridge the gap between double diffusive convection in a Darcy porous medium (low permeability) and double diffusive convection in a classical fluid, and may be helpful in the solidification of a binary alloy.

2. Mathematical Formulation

A sparsely packed fluid-saturated porous medium confined between two parallel horizontal walls has been considered. Cartesian co-ordinates have been taken with the origin in the middle of the porous layer and the z-axis vertically upwards, so that the porous medium lies between the planes $z=-d/2$ and $z=d/2$. The walls are infinitely extended in x- and y-directions, and are rigid. We applied an adverse temperature gradient between the walls of the porous layer by heating from below and cooling from above. Also a stabilizing uniform concentration gradient has been maintained between the walls of the layer.

We apply the following external conditions to modulate the walls' temperatures:

$$T(t)=T_{\mathrm{R}}+\Delta T[1+\varepsilon\mathrm{Re}\{\mathrm{e}^{\mathrm{i}\omega t}\}] \text{ at } z=-d/2, \quad (1a)$$

$$T(t)=T_{\mathrm{R}}+\Delta T\varepsilon\mathrm{Re}\{\mathrm{e}^{\mathrm{i}(\omega t+\phi)}\} \text{ at } z=d/2. \quad (1b)$$

Here T_{R} is the reference temperature, ε represents the amplitude of the modulation, ΔT is the temperature difference, ϕ the phase angle, and ω the modulation frequency. The following three cases are considered: (a) the walls' temperature modulation is in-phase, i. e. $\phi=0$; (b) the temperature modulation is out-of-phase, i. e. $\phi=\pi$; and (c) when only the lower wall's temperature is modulated, the upper wall is held at constant temperature, i. e. $\phi=\mathrm{i}\infty$. Also the boundary conditions on S are

$$S=S_{\mathrm{R}}+\Delta S \text{ at } z=-d/2 \text{ and } S=S_{\mathrm{R}} \text{ at } z=d/2, \quad (2)$$

where S_{R} is the concentration and ΔS is the solute difference across the porous layer. The basic temperature $T_{\mathrm{b}}(z,t)$ and concentration $S_{\mathrm{b}}(z)$ fields are given by

$$\gamma\frac{\partial T_{\mathrm{b}}}{\partial t}=\kappa_T\frac{\partial^2 T_{\mathrm{b}}}{\partial z^2}, \quad (3)$$

$$\frac{\mathrm{d}^2 S_{\mathrm{b}}}{\mathrm{d}z^2}=0, \quad (4)$$

where $\gamma=(\rho c_p)_{\mathrm{m}}/(\rho c_p)_{\mathrm{f}}$ is the heat capacity ratio and $\kappa_T=\kappa_{\mathrm{m}}/(\rho c_p)_{\mathrm{f}}$ is the effective thermal diffusivity, while κ_{m} is the effective thermal conductivity of the porous medium and t is the time. Equation (3) is solved subject to the conditions (1). We write

$$T_{\mathrm{b}}(z,t)=T_{\mathrm{S}}(z)+\varepsilon\mathrm{Re}\{T_O(z,t)\}, \quad (5)$$

where

$$T_{\mathrm{S}}(z)=T_{\mathrm{R}}+\Delta T\left(\frac{1}{2}-\frac{z}{d}\right), \quad (6)$$

$$T_O(z,t)=\frac{\Delta T}{\sinh\lambda}\left\{\mathrm{e}^{\mathrm{i}\phi}\sinh\lambda\left(\frac{1}{2}+\frac{z}{d}\right)+\sinh\lambda\left(\frac{1}{2}-\frac{z}{d}\right)\right\}\mathrm{e}^{\mathrm{i}\omega t}, \quad (7)$$

and

$$\lambda^2=\mathrm{i}\gamma\omega d^2/\kappa_T. \quad (8)$$

Also (4) is solved subject to the conditions (2). We get

$$S_{\mathrm{b}}=S_{\mathrm{R}}+\frac{\Delta S}{2}(1-2z/d). \quad (9)$$

The non-dimensional temperature gradient, $\partial T_{\mathrm{b}}/\partial z$, can be obtained from the dimensionless form of (5) as

$$\frac{\partial T_{\mathrm{b}}}{\partial z}=-1+\varepsilon\mathrm{Re}[g(z)\mathrm{e}^{\mathrm{i}\omega t}], \quad (10)$$

where

$$g(z)=\frac{\lambda}{\sinh\lambda}\left\{\mathrm{e}^{\mathrm{i}\phi}\cosh\lambda\left(\frac{1}{2}+z\right)-\cosh\lambda\left(\frac{1}{2}-z\right)\right\}, \quad (11)$$

and

$$\lambda^2=\mathrm{i}\gamma\omega. \quad (12)$$

Also the non-dimensional form of the concentration gradient from (9) is given by

$$\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}=-1. \quad (13)$$

Using the normal mode technique, dimensionless, the linearized equations for the perturbed variables, namely the vertical component of the velocity w, the temperature θ, and the solute concentration S, are [12]

$$\frac{P_r^{-1}}{\delta}(D^2-a^2)\frac{\partial w}{\partial t} = [(D^2-a^2)^2 - P_l^{-1}(D^2-a^2)]w - a^2R\theta + a^2R_{\mathrm{S}}S, \quad (14)$$

$$\gamma\frac{\partial\theta}{\partial t} = -\left(\frac{\partial T_{\mathrm{b}}}{\partial z}\right)w + (D^2-a^2)\theta, \quad (15)$$

$$\frac{\partial S}{\partial t} = -\left(\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}\right)w + \tau(D^2-a^2)S, \quad (16)$$

where $D \equiv \partial/\partial z$, and $a = (a_x^2 + a_y^2)^{1/2}$ is the horizontal wave number. The non-dimensional parameters which appear in the above equations are the thermal Rayleigh number $R = \frac{\alpha g \Delta T d^3}{\nu\kappa_T}$, the solute Rayleigh number $R_{\mathrm{S}} = \frac{\beta g \Delta S d^3}{\nu\kappa_T}$, the Prandtl number $P_r = \nu/\kappa_T$, the porous parameter $P_l = k/d^2$, and the diffusivity ratio $\tau = \kappa_{\mathrm{S}}/\kappa_T$. g is the acceleration due to gravity, $\nu = (\mu/\rho_{\mathrm{R}})$ the kinematic viscosity, κ_{S} the solute diffusivity, α and β are the coefficients of thermal and solute expansion, respectively, and δ is the porosity of the medium. The following scaling has been used to non-dimensionalize the variables: d for the length, ΔT for the temperature, d^2/κ_T for the time, κ_T/d for the velocity, ΔS for the solute concentration, and κ_T/d^2 for the modulation frequency. Henceforth the values of δ and γ are set equal to one. The relation between the reference density (ρ_{R}), temperature (T_{R}) and concentration (S_{R}) is

$$\rho = \rho_{\mathrm{R}}[1 - \alpha(T - T_{\mathrm{R}}) + \beta(S - S_{\mathrm{R}})]. \quad (17)$$

The boundary conditions for the rigid and conducting walls are given by

$$w = Dw = \theta = S = 0 \quad \text{at} \quad z = \pm\frac{1}{2}. \quad (18)$$

3. Methods

The Galerkin method is used to transform the partial differential equations (14)–(16) into a system of ordinary differential equations. The reduced system of equations will be solved numerically. We put

$$w(z,t) = \sum_{m=1}^{N} A_m(t)\psi_m(z), \quad (19)$$

$$\theta(z,t) = \sum_{m=1}^{N} B_m(t)\varphi_m(z), \quad (20)$$

$$S(z,t) = \sum_{m=1}^{N} C_m(t)\phi_m(z), \quad (21)$$

where

$$\psi_m(z) = \begin{cases} \dfrac{\cosh\mu_m z}{\cosh\frac{\mu_m}{2}} - \dfrac{\cos\mu_m z}{\cos\frac{\mu_m}{2}}, & \text{if } m \text{ is odd},\\[2ex] \dfrac{\sinh\mu_m z}{\sinh\frac{\mu_m}{2}} - \dfrac{\sin\mu_m z}{\sin\frac{\mu_m}{2}}, & \text{if } m \text{ is even}, \end{cases} \quad (22)$$

$$\varphi_m(z) = \sqrt{2}\sin m\pi\left(z + \frac{1}{2}\right), \quad (23)$$

$$\phi_m(z) = \sqrt{2}\sin\left[(m+1)\pi z + (m-1)\frac{\pi}{2}\right] \quad (m = 1,2,3,\ldots). \quad (24)$$

The above functions $\psi_m(z)$, $\varphi_m(z)$ and $\phi_m(z)$ are defined such that they vanish at $z = \pm\frac{1}{2}$, and each forms an orthonormal set in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$. Also, for the derivatives of $\psi_m(z)$ to vanish at the boundaries, it is required that μ_m are the roots of the characteristic equation [35]

$$\tanh\frac{1}{2}\mu_m - (-1)^m\tan\frac{1}{2}\mu_m = 0. \quad (25)$$

We substitute the expressions (19)–(21) into (14)–(16), and then multiply by $\psi_n(z)$, $\varphi_n(z)$ and $\phi_n(z)$ ($n = 1,2,3,\ldots,N$), respectively. After integrating the resulting equations with respect to z in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$, we get a system of $3N$ ordinary differential equations for the unknown coefficients $A_n(t)$, $B_n(t)$ and $C_n(t)$:

$$\begin{aligned} &P_r^{-1}\sum_{m=1}^{N}[K_{nm} - a^2\delta_{nm}]\frac{\mathrm{d}A_m}{\mathrm{d}t} = \\ &\sum_{m=1}^{N}\big[\{(\mu_m^4 + a^4)\delta_{nm} - 2a^2K_{nm}\} \\ &\quad - P_l^{-1}(K_{nm} - a^2\delta_{nm})\big]A_m \\ &- a^2R\sum_{m=1}^{N}P_{nm}B_m + a^2R_{\mathrm{S}}\sum_{m=1}^{N}U_{nm}C_{nm}, \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{\mathrm{d}B_n}{\mathrm{d}t} &= \sum_{m=1}^{N}\big[P_{nm} - \varepsilon\mathrm{Re}\{F_{nm}\mathrm{e}^{\mathrm{i}\omega t}\}\big]A_m \\ &\quad - (n^2\pi^2 + a^2)B_n, \end{aligned} \quad (27)$$

$$\frac{dC_n}{dt} = \sum_{m=1}^{N} U_{mn}A_m - \tau[(n+1)^2\pi^2 + a^2]C_n \quad (n = 1,2,3,\ldots,N), \tag{28}$$

where δ_{nm} is the Kronecker delta. The integrals in the above equations are

$$K_{nm} = \int_{-1/2}^{1/2} \psi_n(z)\mathrm{D}^2\psi_m(z)\mathrm{d}z, \tag{29}$$

$$P_{nm} = \int_{-1/2}^{1/2} \psi_n(z)\varphi_m(z)\mathrm{d}z, \tag{30}$$

$$U_{nm} = \int_{-1/2}^{1/2} \psi_n(z)\phi_m(z)\mathrm{d}z, \tag{31}$$

and

$$F_{nm} = \int_{-1/2}^{1/2} \varphi_n(z)\psi_m(z)g(z)\mathrm{d}z. \tag{32}$$

The above integrals have been evaluated numerically [36]. Also, using the Runge-Kutta-Gill procedure [36], we integrate the above system of equations (26) – (28) and get the fundamental matrix C of solutions. The Rutishauser method [37] is used to obtain the eigenvalues of the matrix C, and the Floquet theory [38] to discuss the stability of the solutions. Then the minimum value of R, known as critical Rayleigh number (R_c), and the corresponding value of a, known as critical wave number (a_c), are been calculated as functions of other parameters.

4. Results and Discussion

During our numerical calculations we found that it is sufficient to take $N = 4$ (four Galerkin terms, two even and two odd). Therefore all the following results are calculated by taking $N = 4$, that is by considering a system of 12 simultaneous ordinary differential equations. The values of the critical Rayleigh number, R_c, and corresponding values of the wave number, a_c, in the absence of modulations ($\varepsilon = 0$) are found as given in Tables 1 – 3.

Now, to calculate the modified value of R_c in the presence of temperature modulation, we consider $\varepsilon \neq 0$ and find the modulation effect on double diffusive convection in a porous medium. To calculate R_c, we consider the following three cases: (a) when the plate temperatures are modulated in-phase, i. e. $\phi = 0$; (b) when the plate temperatures are modulated out-of-phase, i. e. $\phi = \pi$; and (c) when only the bottom plate temperature is modulated, the upper plate is held at a constant temperature, i. e. $\phi = \mathrm{i}\infty$. If the value of R_c is below the x-axis, the effect of modulation is destabilizing, however, if it is above the x-axis, the modulation effect would be stabilizing. Figures 1 – 10 show the variation of the critical Rayleigh number, R_c, with respect to the modulation frequency, ω, for values of the different variables.

Table 1. Results for $R_S = 500.0$; $\tau = 0.05$; $\varepsilon = 0.0$.

No.	P_l	a_c	R_c
1.1	∞	3.104	1750.3
1.2	1.0	3.108	1794.6
1.3	0.1	3.138	2191.7
1.4	0.01	3.229	6099.2

Table 2. Results for $P_l = 1.0$; $\tau = 0.05$; $\varepsilon = 0.0$.

No.	R_S	a_c	R_c
2.1	100.0	3.116	1762.2
2.2	500.0	3.108	1794.6
2.3	1000.0	3.101	1831.6

Table 3. Results for $P_l = 1.0$; $R_S = 500.0$; $\varepsilon = 0.0$.

No.	τ	a_c	R_c
3.1	0.03	3.103	1819.7
3.2	0.05	3.108	1794.6
3.3	0.5	3.117	1757.9

In Figs. 1 – 3, we consider the variation of R_c with ω, for $P_l = 1.0$, 0.1 and 0.01, respectively, the values of the other parameters are: $\varepsilon = 0.4$, $P_r = 1.0$, $\tau = 0.05$, $R_S = 500.0$. On comparing the values of R_c from different figures, corresponding to the respective cases, it is found that, as the value of the porous parameter P_l decreases, the value of R_c increases, indicating that the effect of small values of the porous parameter P_l is to delay the onset of convection. Now, to discuss the effect of temperature modulation on the onset of double diffusive convection, first we consider the case of in-phase modulation. From Figs. 1 – 3, we observe that initially, when ω is small, the effect of modulation is destabilizing as convection occurs at a lower Rayleigh number than that in the unmodulated case (Table 1). The effect of modulation becomes maximum (destabilizing) at around $\omega = 17$, and then stabilizing near $\omega = 60$. However, when ω becomes very large, the effect of modulation disappears alltogether, since the value of R_c approaches the unmodulated value of R_c (Table 1). For out-of-phase temperature modulation or when the upper plate is at constant temperature, it is found that the effect of modulation is maximum (stabilizing) near $\omega = 0$, and becomes less

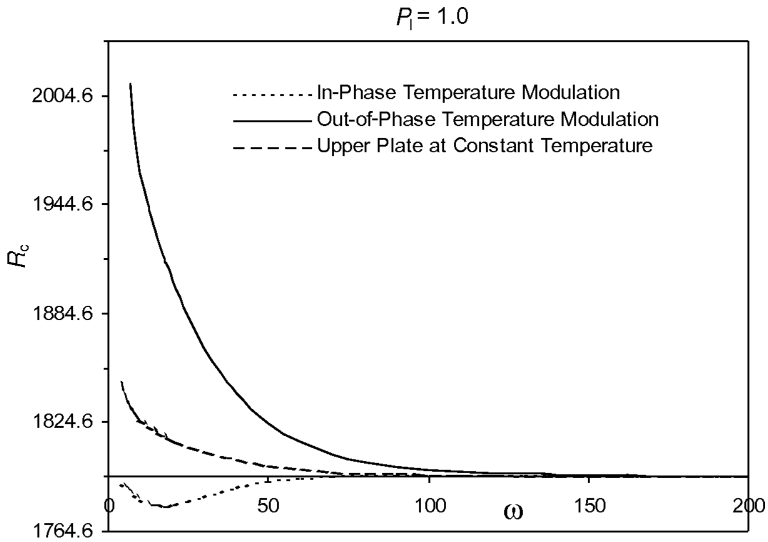

Fig. 1. Variation of R_c with ω. $\varepsilon = 0.4$; $P_r = 1.0$; $\tau = 0.05$; $R_S = 500.0$.

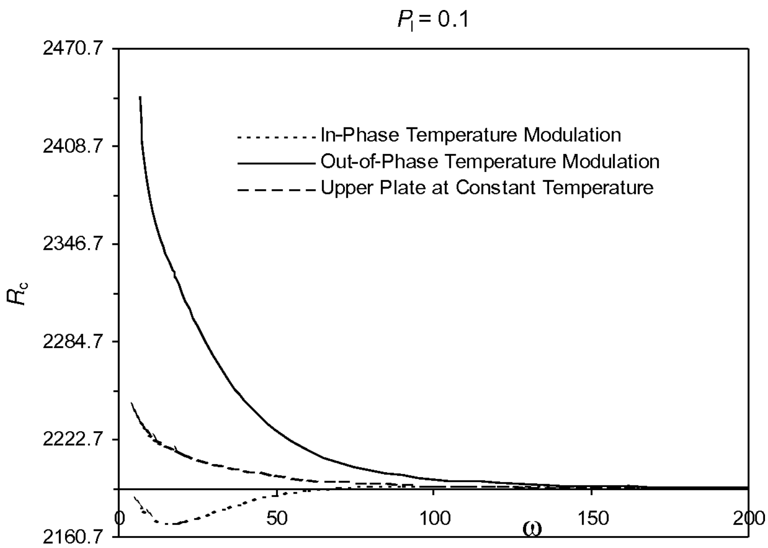

Fig. 2. Variation of R_c with ω. $\varepsilon = 0.4$; $P_r = 1.0$; $\tau = 0.05$; $R_S = 500.0$.

stabilizing for intermediate values of ω. The effect of modulation becomes zero when $\omega \to \infty$. At high frequencies, since the modulation becomes very fast, the temperature in the fluid layer is unaffected by the modulation except for a thin porous layer, so that we find almost the same value of R_c as in the unmodulated case for large values of ω.

In Figs. 4 – 6, we consider the variation of R_c with ω at different values of R_S. The other parameters are: $\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$, $\tau = 0.05$. From the figures we find that an increment in the value of R_S increases the value of the critical Rayleigh number, R_c, which shows that the effect of increasing the solute Rayleigh number, R_S, is to suppress the onset of double diffusive convection, as convection occurs at a higher Rayleigh number. In Figs. 7 – 9 we depict the variation of R_c with ω for different values of the diffusivity ratio, τ. The values of the other parameters are: $\varepsilon = 0.4$, $P_l = 1.0$, $P_r = 1.0$, $R_S = 500.0$. We observe from the figures that an increase in the value of τ decreases the value of R_c. Thus increasing τ favours the onset of double diffusive convection, as convection takes place at an earlier point. Also the effect of modulation on double diffusive convection in the Figs. 4 – 9 for all three cases (a), (b), and (c) is found qualitatively similar to that obtained

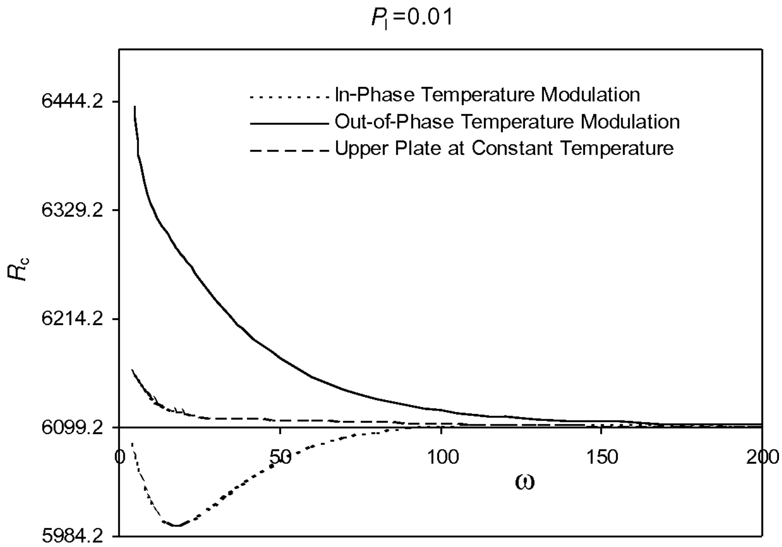

Fig. 3. Variation of R_c with ω. $\varepsilon = 0.4$; $P_r = 1.0$; $\tau = 0.05$; $R_S = 500.0$.

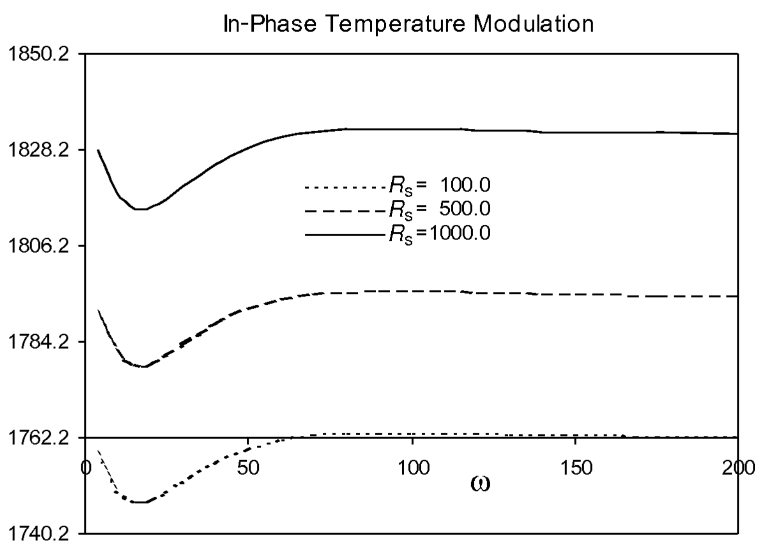

Fig. 4. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $\tau = 0.05$.

in Figs. 1–3. Further, comparing the results of the present study with that of Bhadauria [1], we find that Figs. 4–9 of the present study are similar to Figs. 1–6 of [1] in all respects except that the value of R_c in the present study is higher. This is due to the consideration of double diffusive convection in a porous medium.

In Fig. 10 we consider the variation of R_c with ω for case (c), where the upper plate is at constant temperature, at $\varepsilon = 0.4$, $P_l = 1.0$, $P_r = 1.0$, $R_S = 500.0$, $\tau = 0.05$, and calculate the values of R_c for $N = 4$ and $N = 6$. On comparing the results, it is found that the error in the results corresponding to $N = 4$ and $N = 6$ is very small. Thus our calculations with $N = 4$ are justified.

5. Conclusion

In this study we considered the effect of temperature modulation of rigid-rigid boundaries on double diffusive convection in a sparsely packed porous medium, with the assumptions that disturbances are infinitesimal and the amplitude of the applied temperature field is small. We draw the following conclusions:

1. We find that the effect of the decreasing porous parameter, P_l, is to increase the value of the critical

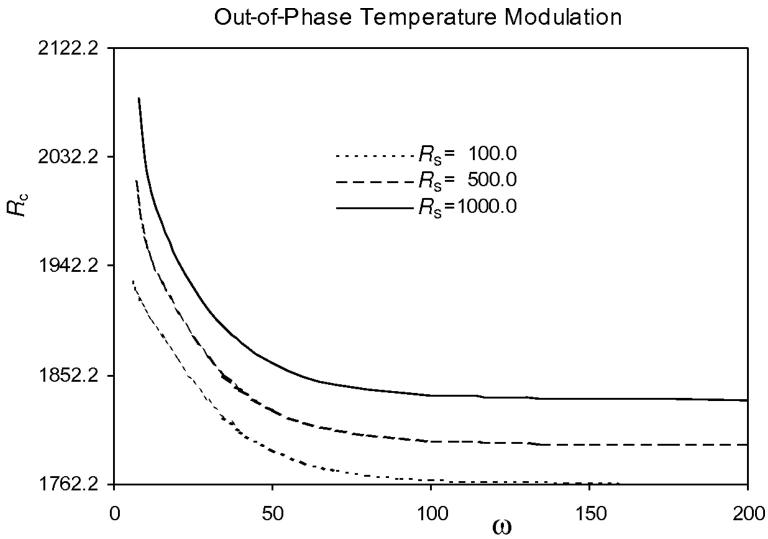

Fig. 5. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $\tau = 0.05$.

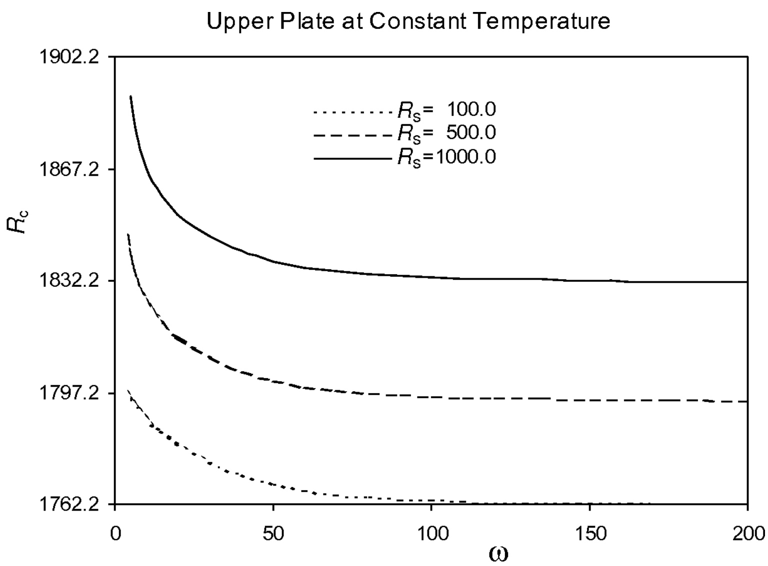

Fig. 6. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $\tau = 0.05$.

Rayleigh number, R_c. Thus the effect of decreasing the permeability is to suppress the onset of double diffusive convection. This is due to the fact that low permeability of the porous medium increases the friction, thereby reduces the convection.

2. The present results, which correspond to Brinkman's model, serve to bridge the gap between the results corresponding to the viscous binary fluid and the Darcy binary fluid in the sense that, when $P_l^{-1} \to 0$ (high permeability), we get the results of a viscous binary fluid [1], and, when $P_l \to 0$ (low permeability), we find Darcy's results for a binary fluid.

3. On increasing the solute Rayleigh number, R_S, the value of R_c increases, thus suppressing the onset of double diffusive convection in porous media. Since salt is a stabilizing agent, increasing R_S would require a higher temperature gradient, thus onset of convection would take place at higher Rayleigh numbers.

4. We find that an increase in the diffusivity ratio, τ, decreases the critical Rayleigh number, thus advancing the onset of double diffusive convection. The diffusivity ratio, τ, increases, when either κ_S increases or κ_T decreases, that is either the solute gradient becomes less stabilizing or the temperature gradient becomes effective at an earlier point. In either of these

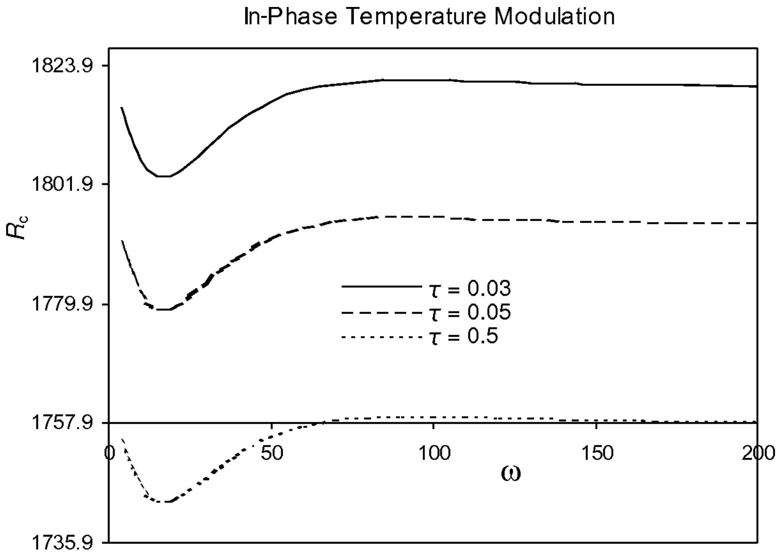

Fig. 7. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $R_S = 500.0$.

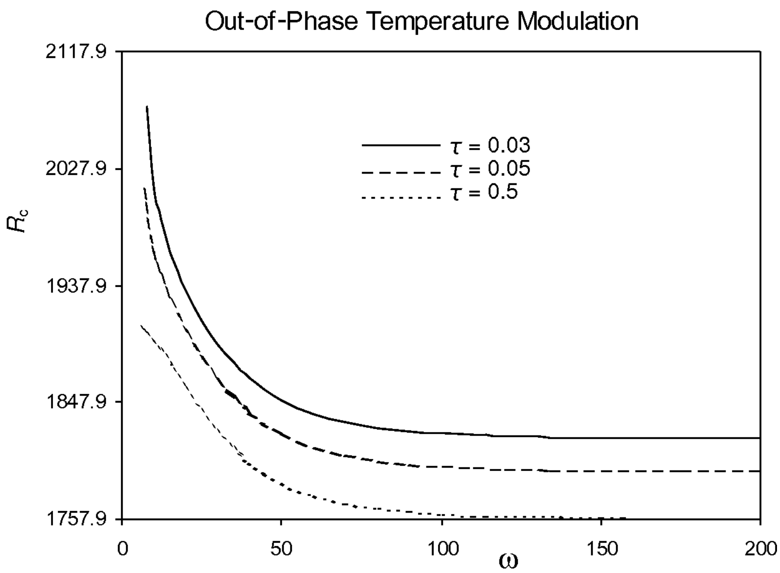

Fig. 8. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $R_S = 500.0$.

two cases the convection occurs at a lower Rayleigh number.

5. For in-phase modulation initially, when ω is small, the modulation effect is also small (destabilizing) and becomes maximum (destabilizing) at around $\omega = 17$. The effect of modulation decreases for intermediate values of ω, becomes stabilizing on further increasing the value of ω, and finally becomes zero when $\omega \to \infty$.

6. For the out-of-phase modulation case or when only the lower wall temperature is modulated, the modulation effect is maximum (stabilizing) near $\omega = 0$, reduces (becomes less stabilizing) for intermediate values of ω, and finally disappears as ω goes to infinity.

7. At high frequency the modulation effect becomes negligible. This is due to the fact that at high frequencies modulation becomes very fast; therefore the temperature in the fluid layer remains unaffected by the modulation except for a thin porous layer. So we find almost the same value of R_c as in the unmodulated case (steady heating).

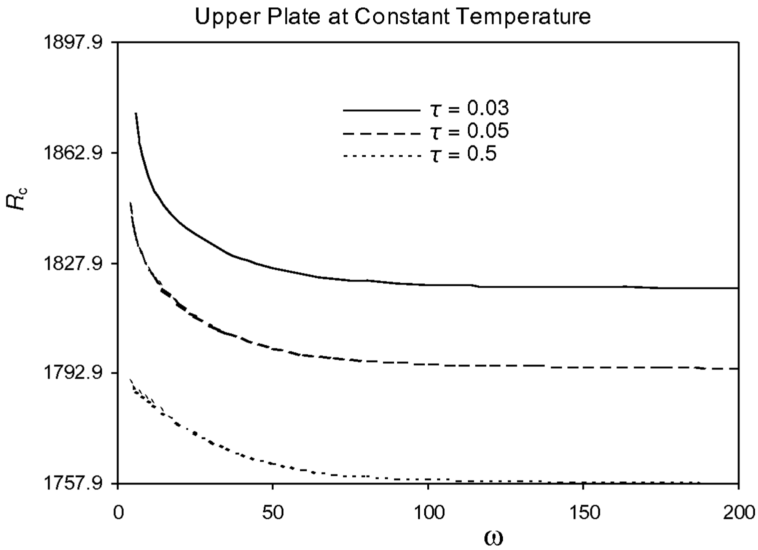

Fig. 9. Variation of R_c with ω. $\varepsilon = 0.4$; $P_l = 1.0$; $P_r = 1.0$; $R_S = 500.0$.

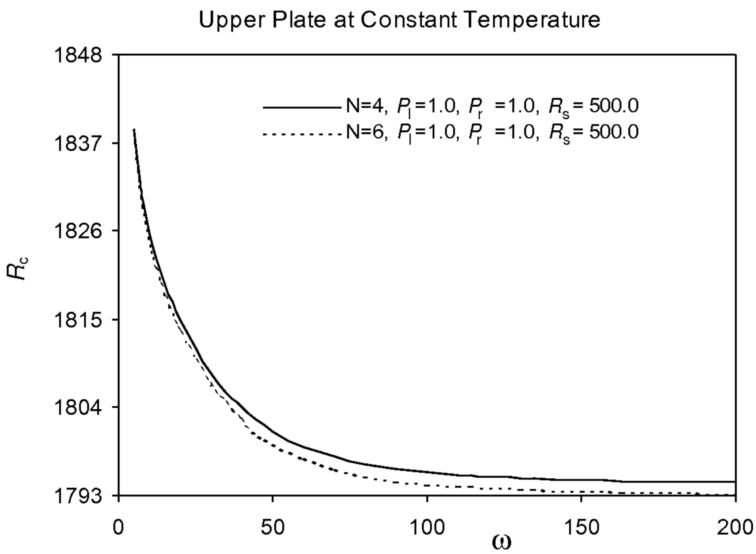

Fig. 10. Variation of R_c with ω. $\varepsilon = 0.4$; $\tau = 0.05$.

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